USN | | | | | | | | | | | **06CS42**

Fourth Semester B.E. Degree Examination, June 2012
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Show that the max number of edges in a simple graph with n vertices is $n(n - 1)/2$ **(06 Marks)**

b. How many vertices will the following graphs have if they contain :

i) 16 edges and all vertices of degree 4.

ii) 12 edges, 6 vertices of degree 3, and other vertices of degree less than 3. **(08 Marks)**

c. Prove that number of vertices of odd degree in a graph is even. **(06 Marks)**

2 a. Determine the number of vertices, number of edges and the number of regions for each of the planar graph in the Fig.Q.2(a)(i) and Fig.Q.2(a)(ii) and also verify that Eulers theorem for connected planar graph is satisfied. **(06 Marks)**

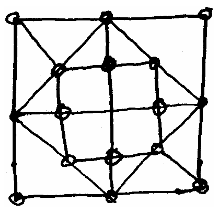

Fig.Q.2(a)(i)

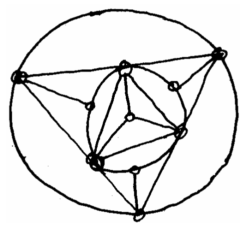

Fig.Q.2(a)(ii)

b. Find a Hamilton cycle, if exists, for each of the graphs shown in Fig.Q.2(b)(i) and Fig.Q.2(b)(ii). If graph has no Hamilton cycle, determine whether it has a Hamilton path. **(06 Marks)**

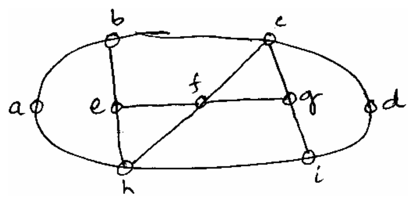

Fig.Q.2(b)(i)

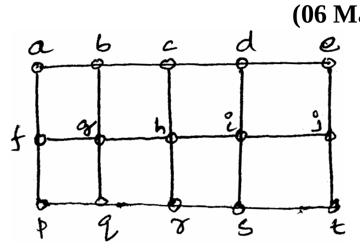

Fig.Q.2(b)(ii)

c. Determine the chromatic polynomials $P(G_n, \lambda)$ and chromatic numbers of each of the following graphs Fig.Q.2(c)(i) and Fig.Q.2(c)(ii). **(08 Marks)**

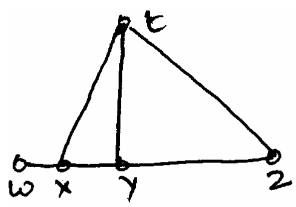

Fig.Q.2(c)(i)

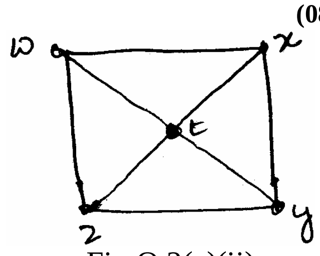

Fig.Q.2(c)(ii)

3 a. Show that for every tree T = (V, E) if $|V| \geq 2$, then T has at least two pendent vertices. **(04 Marks)**

b. Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of pendent vertices in T. **(06 Marks)**

c. What is a prefix code? Construct an optimal prefix code for the symbols a, b, c, …. i, j that occur in a given sample with respect to frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 03. **(10 Marks)**

4 a. Explain Prim's algorithm for finding shortest spanning tree of a weighted graph. **(08 Marks)**

b. Define the following with respect to bipartite graph G = (V, E) :
   i) Complete matching ; ii) Maximal matcing
   ii) Deficiency of graph G. **(06 Marks)**

c. Define : i) Cutset ; ii) Edge-connectivity ; iii) Vertex – connectivity with one example for each. **(06 Marks)**

PART – B

5 a. State two basic counting principles. Give on example for each. **(06 Marks)**

b. A committee of 4 is to be chosen out of 6 Englishman, 5 Frenchmen and 4 Indians, the committee is to contain one of each nationality
   i) In how many ways can it be done?
   ii) In how many arrangements will a particular Indian be? **(08 Marks)**

c. Consider the following program segment, where i, j and k are integer variables

```
for i : = 1 to 20 do
for j : = 1 to i do
for k : = 1 to j do
print(i ∗ j + k).
```

Determine how many times the print( ) statement is executed in above program. **(06 Marks)**

6 a. In how many ways can these integers 1, 2, 3,….., 10 be arranged in a line so that no even integer is in its natural position. **(06 Marks)**

b. Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2, 3, or 5. **(08 Marks)**

c. A board consists of the shaded part as in the Fig.Q.6(c). Find its Rook polynomial. **(06 Marks)**

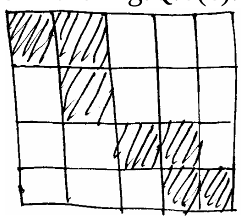

Fig.Q.6(c)

7 a. Determine the coefficient of x^8 in $\dfrac{1}{(x-3)(x-2)^2}$. **(08 Marks)**

b. Find the generating function for each of the following sequence :
   i) 0, 2, 6, 12, 20, 30, 42, ……
   ii) 8, 26, 54, 92,…….. **(06 Marks)**

c. Find a formula to express $0^2 + 1^2 + 2^2 + \ldots.. + n^2$ as a function of n, using summation operator. **(06 Marks)**

8 a. Solve the relation $F_{n+2} = F_{n+1} + F_n$, where $n \ge 0$ and $F_0 = 0, F_1 = 1$. **(06 Marks)**

b. Using generating function, solve $y_{n+2} - 4y_{n+1} + 3y_n = 0$ given that $y_0 = 2, y_1 = 4$. **(08 Marks)**

c. Find the generated solution of $S(K) - 3S(K-1) - 4S(K-2) = 4^K$ where $K \ge 2$. **(06 Marks)**

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